Mark Scheme (Results)

January 2021

Pearson Edexcel IAL Mathematics

Pure Mathematics P3
Paper WMA13 / 01

Question Number	Scheme	Marks
1	$\int \frac{x^2-5}{2x^3}\,dx = \int Ax^{-1} - Bx^{-3}\,dx = C\ln x + Dx^{-2}\,(+c)$	M1 dM1
	$= \frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$	A1
		(3)
		Total 3

M1: Correct **attempt** to integrate.

Score for an attempt to divide by the x^3 term forming a sum of two terms and then integrating.
Award for achieving one term in the correct form. Either $C\ln x + ...$ **or** $... + Dx^{-2}$
Note that $C\ln ax$ and versions such as $k\ln 2x^3$ are also acceptable for $C\ln x$ so look at responses involving lns carefully . Ignore spurious notation e.g. $\int C\ln x$ for the M marks as long as integration has been attempted

dM1: Achieves both terms in the correct form. Score for $\pm C\ln x \pm Dx^{-2}$ or equivalent
Be aware that $C\ln ax \pm Dx^{-2}$ and other variations are also correct

A1: $\frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$ or equivalent simplest form **with the** $+c$. E.g $\ln\sqrt{x} + \frac{5}{4x^2} + c$
ISW after a correct answer.

Some candidates may incorporate the $+c$ within the log so $\frac{1}{2}\ln kx + \frac{5}{4}x^{-2}$ where k is an arbitrary constant is ok.

Note that $\frac{1}{2}\ln 2x + \frac{5}{4}x^{-2} + c$ is not the simplest form and is A0. $\int \frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$ would also be A0

....

Attempts via integration by parts can be scored in the same way

$$\int \frac{x^2-5}{2x^3}\,dx = \int (x^2-5)\times\frac{1}{2}x^{-3}\,dx = (x^2-5)\times -\frac{1}{4}x^{-2} - \int 2x\times -\frac{1}{4}x^{-2}dx = (x^2-5)\times -\frac{1}{4}x^{-2} + \frac{1}{2}\ln x + c$$

M1: For an attempt to integrate by parts the correct way around and achieves $(x^2-5)\times px^{-2} \pm q\ln ax + c$
If the rule is quoted it must be correct.
It is possible to integrate by parts the other way around but unlikely. It can be scored in a similar way.

dM1: Score for

- either then simplifying to an expression of the form $\pm C\ln x \pm Dx^{-2}$ with or without "+ c" which could be numerical
- or integrating to a correct but unsimplified answer $(x^2-5)\times -\frac{1}{4}x^{-2} + \frac{1}{2}\ln ax$ with or without "+ c"

A1: $\frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$ NOT $\frac{1}{2}\ln x + \frac{5}{4}x^{-2} + \underline{\underline{\frac{1}{4} + c}}$ (The answer must be in simplest form and **with the** $+c$)

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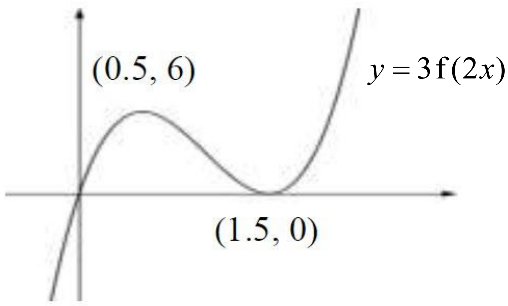
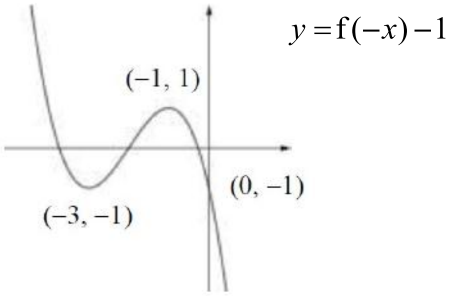
Question Number	Scheme		Marks
2(i)		Shape (two way stretch)	B1
		Maximum at (0.5, 6)	B1
		Minimum at (1.5, 0)	B1
			(3)
(ii)		Shape and position	B1
		Minimum at (−3, −1) and maximum at (−1, 1)	B1
		Crosses y-axis at (0, −1)	B1
			(3)
			Total 6

(i)

B1: Same shape **passing through the origin** with evidence of a two way stretch.
Minimum must be on the x-axis and the graph must be in quadrants 1 and 3
Evidence is $(3,0)\to(a,0)$ where $a\neq 3$ and $(1,2)\to(b,c)$ where $b\neq 1$ and $c\neq 2$
Condone slips of the pen and **mark positively** but the curve should **neither b**end back **significantly** at either end **nor** consist of three straight lines

B1: Maximum at (0.5, 6). Condone a "$\wedge$" shape to the curve here.
There must be a sketch for this to be awarded.
The maximum point may be implied by the sight of 0.5 and 6 being marked on the correct axes in the correct position.

B1: Minimum at (1.5, 0). Condone a "$\vee$ " shape to the curve here.
There must be a sketch for this to be awarded.
Allow this with 1.5 marked on the x-axis (at the minimum point) and condone marked (0, 1.5) on the x-axis

(ii)

B1: Reflection in the y-axis followed by a vertical translation. Look for a $-x^3$ shaped **crossing the *y*-axis but not at the origin** with turning points to the left of the y- axis. Don't be concerned about the coordinates or relative "heights" of the turning points or the y intercept for this mark.
See conditions for shape in (i).

B1: A minimum at (−3, −1) and a maximum at (−1, 1) and at only these points. These may be implied. See part i second B mark. They must be in the correct quadrants and be turnng points, not just points on the curve

B1: Award for a curve crossing the y-axis at (0, −1) . May be awarded for a curve stopping at the y-axis at (0, −1)
There must be a sketch for this to be awarded.
Allow this with −1 marked on the y-axis and condone marked (−1, 0) on the y-axis

Question Number	Scheme	Marks
3(a)	$2x^2-3x-5=(2x-5)(x+1)$	B1
	$3-\dfrac{x-2}{x+1}+\dfrac{5x+26}{2x^2-3x-5}=\dfrac{3(x+1)(2x-5)-(x-2)(2x-5)+5x+26}{(x+1)(2x-5)}$	M1 A1
	$=\dfrac{(4x+1)(x+1)}{(x+1)(2x-5)}=\dfrac{4x+1}{2x-5}$	A1
		(4)
(b)	Correct attempt at inverse $y=\dfrac{4x+1}{2x-5}\Rightarrow x=...$	M1
	$f^{-1}(x)=\dfrac{5x+1}{2x-4}$	A1
		(2)
(c)	$2<x<\dfrac{17}{3}$	M1 A1
		(2)
		Total 8

(a)

B1: Correct factorisation, can be scored anywhere. Sight of $2x^2-3x-5=(2x-5)(x+1)$ Condone $2(x-2.5)(x+1)$

M1: Attempts to combine **all three terms** using a common denominator. Allow the terms to be separate.

There must be an **attempt** to adapt the numerators of the first two terms, one of them must be adapted correctly.

So allow for example $3-\dfrac{x-2}{x+1}+\dfrac{5x+26}{2x^2-3x-5}=\dfrac{3(x+1)(2x-5)-(x-2)\times 2x-5+5x+26}{(x+1)(2x-5)}$

This may be done in stages but is only scored when all three terms are combined.

Condone a fraction where the denominator $(x+1)(2x^2-3x-5)$ is used. (In this case there must be an attempt to adapt the numerators of the all terms and two of the three numerators must be adapted correctly)

$$3-\frac{x-2}{x+1}+\frac{5x+26}{2x^2-3x-5}=\frac{3(x+1)(2x^2-3x-5)}{(x+1)(2x^2-3x-5)}-\frac{(x-2)(2x^2-3x-5)}{(x+1)(2x^2-3x-5)}+\frac{(5x+26)(x+1)}{(x+1)(2x^2-3x-5)}\quad **$$

A1: Correct fraction with denominator $(x+1)(2x-5)$ or equivalent such as $2x^2-3x-5$

Allow this to be given separately

$$3-\frac{x-2}{x+1}+\frac{5x+26}{2x^2-3x-5}=\frac{3(x+1)(2x-5)}{(x+1)(2x-5)}-\frac{(x-2)(2x-5)}{(x+1)(2x-5)}+\frac{5x+26}{(x+1)(2x-5)}$$

If ** was given then they must proceed to $\dfrac{(x+1)(4x^2+5x+1)}{(x+1)(2x^2-3x-5)}$

A1: Correct fraction (or correct values). Proceeds to $\dfrac{4x+1}{2x-5}$ via $\dfrac{(4x+1)(x+1)}{(x+1)(2x-5)}$ oe.

(b)

M1: Attempts at the method for finding the inverse.

Score for an attempt to change the subject for their $y=\frac{ax+b}{cx+d}$ or possibly $y=\frac{a}{c}\pm\frac{e}{cx+d}$

Look for a minimum of cross multiplying by $cx+d$ and proceeding to a form $x=\mathrm{g}(y)$

Some candidates may swap x and y first e.g $x=\frac{ay+b}{cy+d}$ and proceed to y =.. which is fine (same conditions)

Allow this to be scored if one (but not more) of a, b or $d = 0$

Allow this to be scored for candidates who don't finish (a) and attempt to change the subject for $y=\frac{ax+b}{cx+d}$

A1: Correct inverse $\mathrm{f}^{-1}(x)=\frac{5x+1}{2x-4}$ but condone $y=\frac{5x+1}{2x-4}$ and $\mathrm{f}^{-1}=\frac{5x+1}{2x-4}$

Allow other equivalents such as $y=\frac{-5x-1}{4-2x}$, $y=\frac{-5x/2-1/2}{2-x}$ or $y=\frac{5}{2}+\frac{11}{2x-4}$

(c)

M1: For finding one "end" of the domain. Ignore any inequalities.

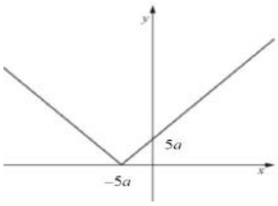
This must be numerical.....and you are just looking for the number, not the variable so y = ... is OK

Sight of either $\frac{17}{3}$ or their $\mathrm{f}(4)$ which may need to be checked

or 2 or $\mathrm{f}(x)$ as $x\to\infty$ that is their $\frac{a}{c}$ (Can be scored for $x\neq 2$)

A1: Correct domain with allowable notation.

Allowable equivalent forms are e.g. $\left(2,\ \frac{17}{3}\right)$, $x>2$ and $x<\frac{17}{3}$. Condone "or"

Question Number	Scheme	Marks
4(a)	Either $x=-\frac{a}{3}$ or $y=a$	B1
	Correct coordinates $\left(-\frac{a}{3}, a\right)$	B1
		(2)
(b)	 V shape in Quad 1 and 2 with the vertex on the negative x-axis	B1
	Intersects/meets axes at $(0, 5a)$ and $(-5a, 0)$	B1
		(2)
(c)	Attempts to solve a correct equation **E.g. either** $x+5a=-3x-a+a \Rightarrow x=...$ **or** $x+5a=3x+a+a \Rightarrow x=...$	M1
	$x=-\frac{5}{4}a$ **or** $x=\frac{3}{2}a$	A1
	$x=-\frac{5}{4}a$ and $x=\frac{3}{2}a$	A1
	$x=... \Rightarrow y=...$ using either equation	dM1
	$\left(-\frac{5}{4}a, \frac{15}{4}a\right)$ and $\left(\frac{3}{2}a, \frac{13}{2}a\right)$	A1
		(5)
		Total 9

(a)

B1: One of $x=-\frac{a}{3}$ **or** $y=a$

B1: Both coordinates correct. Allow this to be written separately as $x=-\frac{a}{3}$ **and** $y=a$

(b)

B1: Correct shape and position in both quadrant 1 and quadrant 2. Condone an asymmetric graph but the vertex must be on the negative x - axis. Condone a free hand sketch as long as the intention was to have two straight lines. Ignore any "dotted" lines

B1: Correct intercepts given as coordinates or as marked in the scheme. Must be on the correct axes in the correct positions. Condone for example (5a, 0) for (0, 5a) if marked on the correct axis. Additional intersections is B0

If the graph is **only** in quadrant 2 this can be scored for meeting the axes at (– 5a, 0) and (0, 5a)

(c)

M1: Attempts to solve either of the correct equations. (Ones that don't involve a modulus)

Allow $x+5a=-3x$ for $x+5a=-3x-a+a$ and $x+5a=3x+2a$ for $x+5a=3x+a+a$

Do not condone attempts where a candidate incorrectly combines the modulus terms even if it leads to a correct value for x. E.g. $|3x+a|-|x+5a|=-a \Rightarrow |2x|=3a \Rightarrow$ scores M0

A1: One correct value for x. Either $x=-\frac{5}{4}a$ or $x=\frac{3}{2}a$ (following a correct non modulus equation)

Allow this mark even if the candidate subsequently rejects the solution

A1: Both values correct $x=-\frac{5}{4}a$ and $x=\frac{3}{2}a$ with no additional values given

This cannot be scored if the candidate rejects either of these solutions

dM1: Correct method to obtain at least one y value using either equation in y.

Evidence would be embedded values leading to y = which may be unsimplified or a correct calculation for their x

A1: Both sets of coordinates $\left(-\frac{5}{4}a, \frac{15}{4}a\right)$ and $\left(\frac{3}{2}a, \frac{13}{2}a\right)$ which may be given x =.., y =... with no extras.

Special case: B1 For either of the two correct coordinates which may be scored with no working or following an attempt that incorrectly combines the modulus terms. Scored 1 0 0 0 0

Question Number	Scheme	Marks
5(a)	$18 = A - 180 \times 1 \Rightarrow A = \ldots$	M1
	$A = 198$	A1
		(2)
(b)	$90 = 198 - 180\mathrm{e}^{-5k} \Rightarrow 180\mathrm{e}^{-5k} = 108$	M1, A1
	$\mathrm{e}^{-5k} = \dfrac{108}{180} \Rightarrow -5k = \ln 0.6 \Rightarrow k = \ldots$	dM1
	$k = -\dfrac{1}{5}\ln\dfrac{3}{5}$ or $k = \dfrac{1}{5}\ln\dfrac{5}{3}$	A1
		(4)
(c)	$\theta = 198 - 180\mathrm{e}^{-9\times\frac{1}{5}\ln\frac{5}{3}} \Rightarrow \theta = \ldots$	M1
	$\theta = 126°\mathrm{C}$ (awrt)	A1
		(2)
(d)	$\left\{\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right\} = -180\left(-\dfrac{1}{5}\ln\dfrac{5}{3}\right)\mathrm{e}^{-\frac{t}{5}\ln\frac{5}{3}} = \ldots$	B1ft
	$\left\{\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right\} = -180\left(-\dfrac{1}{5}\ln\dfrac{5}{3}\right)\mathrm{e}^{-\frac{9}{5}\ln\frac{5}{3}} = \ldots$	M1
	$= 7.33$ °C min^{-1} (awrt)	A 1
		(3)
		Total 11

(a)
M1: Substitutes $\theta = 18$ and $t = 0$ uses $e^0 = 1$ and proceeds to find a value for A. Look for $18 = A - 180 \times 1 \Rightarrow A = ...$
A1: $A = 198$. Condone $A = 198°C$. Sight of 198 is sufficient to award both marks.
(b)
M1: Substitutes $\theta = 90$ and $t = 5$ with their value for A and proceeds to an equation of the form $Pe^{\pm 5k} = Q$
A1: Correct equation $180e^{-5k} = 108$ o.e.

dM1: Correct order of operations using ln's to make k the subject. Do not award if taking log₁₀'s. ($\log\frac{3}{5} = -0.22..$)

$$Pe^{\pm 5k} = Q \Rightarrow \pm 5k = \ln\left(\frac{P}{Q}\right) \Rightarrow k = ... \text{ with } \frac{P}{Q} > 0 \quad \text{OR } Pe^{\pm 5k} = Q \Rightarrow \ln P \pm 5k = \ln Q \Rightarrow k = ... \text{ with } P, Q > 0$$

It would be implied by a decimal equivalent to 3sf. So for correct values accept $k =$ awrt 0.102

A1: Cao. Allow equivalent correct exact answers in the required form. E.g. $k = -0.2\ln 0.6$ and $k = \frac{1}{5}\ln\frac{180}{108}$
(c)
M1: Substitutes their A and their k with $t = 9$ to find a value for θ.
Sight of embedded values is sufficient evidence and condone sign slips which may be common.
A1: Awrt 126°C but condone a lack of units. Sight of awrt 126 following $k =$ awrt 0.102 is sufficient to award both marks.
(d)
B1ft: Correct differentiation. Follow through their k (even as a decimal or as "k"). Allow for $180ke^{-kt}$
M1: Substitutes $t = 9$ into a function of the form $\left(\frac{d\theta}{dt} =\right)\beta e^{-kt}$ following through on their k. Condone $\frac{d\theta}{dt} = \frac{dy}{dx}$
A1: Awrt 7.33 Ignore units.
Award B0 M1 A1 for candidates who achieve $\pm$ awrt 7.33 following a sign error in their $\frac{d\theta}{dt}$

....

Answers without working in (d).
The rubric states that candidates are required to show sufficient working to make their method clear.
They must have a value of k to do this part.
SC: B1ft scored for

- either stating $\frac{d\theta}{dt}$ at $t = 9$ is .... You will need check. Allow accuracy to 3sf and ft on their k
- or following correct k just writing down awrt 7.33

....

Question Number	Scheme	Marks
6(a)	$\left(\text{f}'(x)\right)=\cos\left(\frac{x}{3}\right)-\frac{1}{3}x\sin\left(\frac{x}{3}\right)$	M1 A1
		(2)
(b)	$\text{f}'(x)=0\Rightarrow\cos\left(\frac{x}{3}\right)-\frac{1}{3}x\sin\left(\frac{x}{3}\right)=0\Rightarrow 1-\frac{1}{3}x\tan\left(\frac{x}{3}\right)=0$	M1
	$\tan\left(\frac{x}{3}\right)=\frac{3}{x}\Rightarrow\quad x=3\arctan\left(\frac{3}{x}\right)$ *	A1*
		(2)
(c)	$x_2=3\arctan\left(\frac{3}{2.5}\right)=\text{awrt } 2.6$	M1
	$x_2=\text{awrt } 2.628$ and $x_6=\text{awrt } 2.586$	A1
		(2)
(d)	$\text{f}'(2.5815)=\cos\left(\frac{2.5815}{3}\right)-\frac{1}{3}(2.5815)\sin\left(\frac{2.5815}{3}\right)=-0.000345...$ $\text{f}'(2.5805)=\cos\left(\frac{2.5805}{3}\right)-\frac{1}{3}(2.5805)\sin\left(\frac{2.5805}{3}\right)=0.000346...$ Chooses a suitable interval and attempts both values	M1
	(Both values correct) Sign change and continuous, therefore root	A1
		(2)
		Total 8

(a)

M1: Attempts the product rule and obtains a derivative of the form $\alpha\cos\left(\frac{x}{3}\right)\pm\beta x\sin\left(\frac{x}{3}\right)$

If the rule is stated or implied to be $vu'-uv'$ it is M0

A1: $\cos\left(\frac{x}{3}\right)-\frac{1}{3}x\sin\left(\frac{x}{3}\right)$ which may be unsimplified

(b)

M1: Sets their $\text{f}'(x)=\alpha\cos\left(\frac{x}{3}\right)\pm\beta x\sin\left(\frac{x}{3}\right)=0$ and proceeds to an equation involving $\tan\left(\frac{x}{3}\right)$

A1*: CSO Proceeds to $x=3\arctan\left(\frac{3}{x}\right)$ following an intermediate line of $\tan\left(\frac{x}{3}\right)=\frac{3}{x}$ or $\tan\left(\frac{x}{3}\right)=\frac{1}{\left(\frac{x}{3}\right)}$

Do not condone $\tan^{-1}$ notation unless correct notation is also given.

(c)

M1: Uses a formula of the type $x=\alpha\arctan\left(\beta\times\frac{1}{x}\right)$ with the 2.5 to find the value of x_2 correct to one dp.

So when $x=3\arctan\left(\frac{3}{x}\right)$ M1 is scored for $x_2=$ awrt 2.6

and when $x=3\arctan\left(\frac{1}{x}\right)$ M1 is scored for $x_2=$ awrt 1.1

A1: $x_2=\text{awrt } 2.628$ and $x_6=\text{awrt } 2.586$

(d)

M1: Chooses a suitable function for **their** $f'(x)=0$ and attempts its value at both 2.5815 and 2.5805.

For the attempt we need to see embedded values as in scheme or one value correct to 1sf for their $f'(x)$

Allowable functions are $f'(x)=\cos\left(\frac{x}{3}\right)-\frac{1}{3}x\sin\left(\frac{x}{3}\right)$ and multiples of this. See scheme.

Follow through on their $\cos\left(\frac{x}{3}\right)-\frac{1}{3}x\sin\left(\frac{x}{3}\right)$

Allow the function to be stated as just $f'(x)$ (or mistakenly written as $f(x)$) as long as one of $f'(2.5805)=...$ or $f'(2.5815)=...$ is correct to 1sf

Other functions are possible, for example $h(x)=x-3\arctan\left(\frac{3}{x}\right)$ and multiples of this.

Follow through on their $x-k\arctan\left(\frac{k}{x}\right)$ Unlikely, but it is also acceptable to pick a tighter interval

A1: Requires correct differentiation and

- both values correct (rounded or truncated to 1sf)
  Note that $h(2.5815)=0.000786...,h(2.5805)=-0.000788...$
- a valid reason that includes both a reference to the sign change and continuity. Condone a mention of continuity of $f(x)$ instead of $f'(x)$
- a minimal conclusion which could be ✓, QED, root.

Question Number	Scheme	Marks
7(a)	Uses $\sin 2x = 2\sin x\cos x$ **AND** $\cos 2x = 1-2\sin^2 x$ o.e. in $\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}$	M1
	$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{2\sin x\cos x}{\cos x}+\frac{1-2\sin^2 x}{\sin x}$ $=\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}+\frac{1}{\sin x}-\frac{2\sin^{\cancel{2}} x}{\cancel{\sin x}} = \frac{1}{\sin x}=\text{cosec } x*$	dM1 A1*
		(3)
(b)	Uses part (a) $\Rightarrow 7+\text{cosec } 2\theta = 3\cot^2 2\theta$	B1
	Either Uses $\pm1\pm\cot^2 2\theta = \pm\text{cosec}^2 2\theta \;\rightarrow$ 3TQ in $\text{cosec } 2\theta$ **Or alternatively** replaces $\text{cosec}2\theta$ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta/\sin^2 2\theta$, multiplies by $\sin^2 2\theta$ and uses $\pm\cos^2 2\theta = \pm1\pm \sin^2 2\theta \;\rightarrow$ 3TQ in $\sin 2\theta$	M1
	$3\text{cosec}^2 2\theta - \text{cosec } 2\theta - 10 = 0$ **or** $10\sin^2 2\theta + \sin 2\theta - 3 = 0$	A1
	$(3\text{cosec } 2\theta+5)(\text{cosec } 2\theta-2)=0$ **or** $(5\sin 2\theta+3)(2\sin 2\theta-1)=0$ $\Rightarrow \text{cosec } 2\theta = -\frac{5}{3}, 2$ **or** $\Rightarrow \sin 2\theta = -\frac{3}{5}, \frac{1}{2}$ $\Rightarrow \sin 2\theta = -\frac{3}{5}, \frac{1}{2} \Rightarrow \theta = ...$	dM1
	$\theta = \frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$ (awrt these values)	A1, A1
		(6)
		Total 9

(a)

M1: Uses

- $\sin 2x = 2\sin x\cos x$
- AND $\cos 2x = 1-2\sin^2 x$ or equivalent. Condone sign slips on the versions of $\cos 2x$

in an attempt to write $\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}$ as an expression in $\sin x$ and $\cos x$

dM1: Adopts a valid approach that can be followed and completes the proof .

All necessary steps may not be shown and condone errors such as writing cos for $\cos x$ or $\sin x^2$ for $\sin^2 x$

A1*: Correct proof showing all necessary intermediate steps with no errors (seen within the body of the solution) or omissions of any of the steps shown. The LHS starting point does not need to be seen

See main mark scheme and below for examples showing all steps and scoring full marks

$$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{\sin x\sin 2x+\cos x\cos 2x}{\sin x\cos x}$$
$$=\frac{2\sin^2 x\cos x+\cos x(1-2\sin^2 x)}{\sin x\cos x}$$
$$=\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}$$
$$=\text{cosec}x$$

$$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}+\frac{\cos 2x}{\sin x}$$
$$=\frac{2\sin^2 x+(\cos^2 x-\sin^2 x)}{\sin x}$$
$$=\frac{\sin^2 x+\cos^2 x}{\sin x}=\frac{1}{\sin x}$$
$$=\text{cosec}x$$

Alt part (a)

M1: For using compound angle formula $\sin x \sin 2x + \cos x \cos 2x = \cos\left(2x - x\right)$

dM1: As in the main scheme, it is for adopting a valid approach that can be followed and completing the proof

A1: Correct proof showing all necessary steps (See below) with no errors or omissions

$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} = \frac{\sin x \sin 2x + \cos x \cos 2x}{\sin x \cos x}$$
$$= \frac{\cos(2x - x)}{\sin x \cos x}$$
$$= \frac{\cancel{\cos x}}{\sin x \cancel{\cos x}}$$
$$= \text{cosec} x$$

....

(b)

B1: States $7 + \text{cosec } 2\theta = 3\cot^2 2\theta$ or exact equivalent which may be implied by subsequent work

OR $7 + \text{cosec } x = 3\cot^2 x$ with $x = 2\theta$

Watch for and do not allow $7 + \text{cosec } \theta 2\theta 3 \cot^2$

M1: Attempts to use part (a) and uses $\pm 1 \pm \cot^2 2\theta = \pm \text{cosec}^2 2\theta$ to form a 3TQ in $\text{cosec } 2\theta$

Condone $3\cot^2 2\theta$ being replaced by $3 \times \pm\text{cosec}^2 2\theta \pm 1$ with or without the bracket.

Condone when the "7" is missing but these attempts will score a maximum of 2 marks. This mark and dM1

The terms don't need to be collected for this mark.

Alternatively replaces cosec2θ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta / \sin^2 2\theta$ within an equation of the form $a + b\text{cosec } 2\theta = c \cot^2 2\theta$ multiplies by $\sin^2 2\theta$ and uses $\pm\cos^2 2\theta = \pm 1 \pm \sin^2 2\theta$ $\rightarrow$ 3TQ in $\sin 2\theta$

A1: Correct equation $3\text{cosec}^2 2\theta - \text{cosec } 2\theta - 10 = 0$ **or** $10\sin^2 2\theta + \sin 2\theta - 3 = 0$

The = 0 may be implied by further work, e.g solution of the equation

Allow this mark even for the correct equation in a different forms. E.g. $3\text{cosec}^2 2\theta - \text{cosec } 2\theta = 10$

dM1: For a correct attempt to solve their 3TQ $\sin 2\theta$ or cosec 2θ leading to a value for θ

If they state that $\sin \theta = -\frac{3}{5}, \frac{1}{2}$ and do not proceed to take arcsin and $\div 2$ it is M0

A1: For two of awrt $\theta = \frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$

A1: For awrt $\theta = \frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$ with no additional values within the range.

....

If you see other worthwhile solutions and the scheme cannot be applied, e.g *t* formula, please send to review

....

How to mark when other variables are used, e.g. $x = 2\theta$

B1: $7 + \text{cosec } x = 3\cot^2 x$

M1: Uses $\pm 1 \pm \cot^2 x = \pm\text{cosec}^2 x$ to form 3TQ in $\text{cosec } x$ ....or the equivalent in $\sin x$

A1: Correct equation $3\text{cosec}^2 x - \text{cosec } x - 10 = 0$ or $10\sin^2 x + \sin x - 3 = 0$

ddM1: For this to be scored there must be an attempt to halve the values, otherwise M0.

Allow full marks to be scored for a candidate who uses a different variable correctly and reaches 4 correct answers

....

Question Number	Scheme	Marks
8(a)	States $\log a = 0.68$ or $\log b = 0.09$	M1
	$a = 4.79$ **or** $b = 1.23$	A1
	States $\log a = 0.68$ and $\log b = 0.09$	M1
	$a = 4.79$ **and** $b = 1.23$ **CSO**	A1
		(4)
(b)	The percentage of the population with access to the internet at the start of 2005	B1
		(1)
(c)	$P = 4.79 \times 1.23^{10} =$ awrt 38	M1, A1
		(2)
		Total 7

(a)

M1: Either states any of $\log a = 0.68$, $a = 10^{0.68}$, $a =$ awrt 4.8

or any of $\log b = 0.09$, $b = 10^{0.09}$, $b =$ awrt 1.2

A1: Achieves either $a =$ awrt 4.79 **or** $b =$ awrt 1.23

M1: States a correct equation for both a and b. See first M mark

A1: Achieves $a = 4.79$ **and** $b = 1.23$ **with no incorrect work**.

Implied by $P = 4.79 \times 1.23^t$ **with no incorrect work**

These are NOT awrt values

Examples of incorrect work are

- $P = ab^t \Rightarrow \log P = \log a \times t \log b$
- $\log P = 0.68 + 0.09t \Rightarrow P = 10^{0.68} + 10^{0.09t} \Rightarrow P = 4.79 \times 1.23^t$

(b)

B1: A correct interpretation. The emboldened words must be present or stated in a similar way

"The **percentage** of the population with access to the **internet** at the start of **2005"**

A minimal answer is "the percentage with access to the internet in 2005"

Also allow "the initial percentage with internet access".

It is acceptable to state 4.79% of the population had access to the in 2005

(c)

M1: For attempting 4.79×1.23^{10} following through on their 4.79 and 1.23, (Ignore subsequent work)

Alternatively attempting $\log P = 0.68 + 10 \times 0.09 \Rightarrow P = ...$

Condone an attempt at 4.79×1.23^{11}

A1: AWRT 38. ISW after sight of awrt 38 and condone misinterpretations such as stating 38 people.

Question Number	Scheme	Marks
9(i)	$\int \frac{3x-2}{3x^2-4x+5}\,dx = \frac{1}{2}\ln\left(3x^2-4x+5\right)(+c)$	M1, A1
		(2)
(ii)	$\int \frac{e^{2x}}{\left(e^{2x}-1\right)^3}\,dx = -\frac{1}{4}\left(e^{2x}-1\right)^{-2}(+c)$	M1, A1
		(2)
		Total 4

(i)

M1: Integrates to a form $\alpha\ln\left(3x^2-4x+5\right)$ where α is a constant . Condone a missing bracket.

Do not accept $\alpha\ln\left(3x^2-4x+5\right)+f(x)$, e.g. $\ln\left(3x^2-4x+5\right)+2x$

If the substitution $u=3x^2-4x+5$ is attempted, the mark can be awarded for $k\ln u$

It is unlikely but $\alpha\ln\beta\left(3x^2-4x+5\right)$ and $\alpha\ln\left(3x^2-4x+5\right)^{\beta}$ are also correct

A1: $\frac{1}{2}\ln\left(3x^2-4x+5\right)$ o.e. with or without the + c. A bracket or modulus must be present.

ISW after a correct answer.

Do not penalise $\frac{\ln\left(3x^2-4x+5\right)}{2}$ or $\ln\left(3x^2-4x+5\right)/2$ if the intention is clear

Penalise spurious incorrect notation for the A mark only. So do not allow $\frac{\ln\left(3x^2-4x+5\right)dx}{2}$

(ii)

M1: Integrates to a form $\beta\left(e^{2x}-1\right)^{-2}$ where β is a constant .

Do not accept $\beta\left(e^{2x}-1\right)^{-2}+g(x)$, e.g. $\beta\left(e^{2x}-1\right)^{-2}+e^{2x}$

Allow substitutions. So for example,

if the substitution $u=e^{2x}-1$ is attempted, the mark can be awarded for ku^{-2}

if the substitution $u=e^{x}$ is attempted, the mark can be awarded for $k\left(u^2-1\right)^{-2}$

if the substitution $u=e^{2x}$ is attempted, the mark can be awarded for $k\left(u-1\right)^{-2}$

A1: $-\frac{1}{4}\left(e^{2x}-1\right)^{-2}$ or exact equivalent with or without the + c

ISW after a correct answer. Need not be simplified

Penalise spurious incorrect notation for the A mark only. So do not allow $\int -\frac{1}{4}\left(e^{2x}-1\right)^{-2}$

Question Number	Scheme	Marks
10(a)	$\frac{dx}{dy}=12\sec^2 2y\tan 2y$	M1, A1
		(2)
(b)	$\frac{dx}{dy}=12\left(\frac{x}{3}\right)\sqrt{\sec^2 2y-1}\Rightarrow\frac{dx}{dy}=12\left(\frac{x}{3}\right)\sqrt{\frac{x}{3}-1}$	M1, A1ft
	$\frac{dy}{dx}=\frac{\sqrt{3}}{4x\sqrt{x-3}}$	A1
		(3)
(c)	$y=\frac{\pi}{12}\Rightarrow x=4$	B1
	$\frac{dx}{dy}=12\times\frac{4}{3}\times\frac{1}{\sqrt{3}}$ or $\frac{dy}{dx}=\frac{1}{12\left(\frac{4}{3}\right)\sqrt{\frac{4}{3}-1}}$	M1
	Correct $m_N=-\frac{16}{\sqrt{3}}$ o.e	A1
	$y-\frac{\pi}{12}=-\frac{16}{\sqrt{3}}(x-4)$	dM1
	$y=-\frac{16\sqrt{3}}{3}x+\frac{64\sqrt{3}}{3}+\frac{\pi}{12}$	A1
		(5)
		Total 10

(a)

M1: Differentiates to a form on the rhs of $\alpha\sec^2 2y\tan 2y$ which may be written $...\sec 2y\times...\sec 2y\tan 2y$

Note that the same scheme can also be applied to students who adapt $x=3\sec^2 2y$ to $x=\pm3\tan^2 2y\pm3$

A1: $\frac{dx}{dy}=12\sec^2 2y\tan 2y$. If the lhs is included it must be correct. So $\frac{dy}{dx}=12\sec^2 2y\tan 2y$ is M1 A0

Condone this to be unsimplified $\frac{dx}{dy}=6\sec 2y\times 2\sec 2y\tan 2y$ ISW after sight of correct answer

(b)

M1: For an attempt to

- replace $\sec^2 2y$ with αx
- use the identity $\pm1\pm\tan^2 2y=\pm\sec^2 2y$ and replaces $\tan 2y=b\sqrt{\pm1\pm d\,x}$

to obtain an expression for $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of x only.

The expression in part (a) must have had $\frac{dx}{dy}$ as a function of both $\sec 2y$ and $\tan 2y$ o.e.

A1ft: Requires a substitution of both $\sec^2 2y$ with $\frac{x}{3}$ and $\tan 2y=\sqrt{\frac{x}{3}-1}$ to obtain a correct expression for $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of x. Follow through on their $\frac{dx}{dy}=\alpha\sec^2 2y\tan 2y$

For a correct $\frac{dx}{dy}$ it is awarded for $\frac{dy}{dx}=\frac{1}{4x\sqrt{\frac{1}{3}x-1}}$

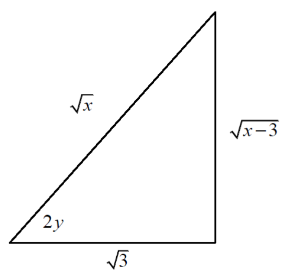
A1: Correct answer in the required form.

Allow equivalents e.g. $\frac{3\sqrt{3}}{12x\sqrt{x-3}}$. Form required is $\frac{p}{qx\sqrt{x-3}}$ where p is irrational and q is an integer

Alt method for (a) and (b) which can be marked in a similar way

(a) M1 A1: $x=3(\cos 2y)^{-2} \Rightarrow \frac{dx}{dy}=12(\cos 2y)^{-3}\sin 2y$

(b) If $x=3(\cos 2y)^{-2} \Rightarrow \cos 2y=\frac{\sqrt{3}}{\sqrt{x}}$



Score in a similar way to the main scheme

M1 A1: $\frac{dx}{dy}=\frac{12\sin 2y}{(\cos 2y)^3}=\frac{12\sqrt{\frac{x-3}{x}}}{\left(\sqrt{\frac{3}{x}}\right)^3}$

Alt (b) via arccos

$$y=\frac{1}{2}\arccos\sqrt{\frac{3}{x}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\frac{1}{\sqrt{1-\left(\sqrt{\frac{3}{x}}\right)^2}}\times\frac{\sqrt{3}}{2}x^{-\frac{3}{2}}$$

M1: For $\frac{dy}{dx}=\lambda\frac{1}{\sqrt{1-\left(\sqrt{\frac{\alpha}{x}}\right)^2}}\times -x^{-\frac{3}{2}}$ A1: Correct and unsimplied A1: Correct and in the required form

(c)

B1: Correct value for x

M1: Attempts to find the value of $\frac{dx}{dy}$ using their part (a) with $y=\frac{\pi}{12}$

the value of $\frac{dy}{dx}$ from an inverted $\frac{dx}{dy}$ using their part (a) with $y=\frac{\pi}{12}$

or the value of $\frac{dy}{dx}$ using their part (b) with their value of x found using $y=\frac{\pi}{12}$.

These may be called m or f ' and not identified as $\frac{dx}{dy}$ or $\frac{dy}{dx}$

A1: Correct normal gradient

dM1: Attempt at the equation of normal at $y=\frac{\pi}{12}$.

The gradient should be either an attempt at the value of $-\frac{dx}{dy}$ at $y=\frac{\pi}{12}$ for their $\frac{dx}{dy}$

or an attempt at the negative reciprocal of their $\frac{dy}{dx}$ at their "4" which must have been found from $y=\frac{\pi}{12}$

A1: Fully correct equation in the required form. ISW after a correct answer

Allow equivalent exact forms e.g. $y=-\frac{16}{\sqrt{3}}x+\frac{64}{\sqrt{3}}+\frac{\pi}{12}$, $y=-\frac{16}{\sqrt{3}}x+\frac{256\sqrt{3}+\pi}{12}$